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# Implementing thinking aloud pair and Pólya problem solving strategies in fractions

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**Abstract.** This study implemented two pedagogical strategies, the Thinking Aloud Pair Problem Solving and Pólya's Problem Solving, to support students' learning of fractions. The participants were 51 students (ages 11-13) from two Year 7 classes in a government secondary school in Brunei Darussalam. A mixed method design was employed in the present study, with data collected from the pre- and post-tests, problem solving behaviour questionnaire and interviews. The study aimed to explore if there were differences in the students' problem solving behaviour before and after the implementation of the problem solving strategies. Results from the Wilcoxon Signed Rank Test revealed a significant difference in the test results regarding student problem solving behaviour,  $z = -3.68$ ,  $p = .000$ , with a higher mean score for the post-test ( $M = 95.5$ ,  $SD = 13.8$ ) than for the pre-test ( $M = 88.9$ ,  $SD = 15.2$ ). This implied that there was improvement in the students' problem solving performance from the pre-test to the post-test. Results from the questionnaire showed that more than half of the students increased scores in all four stages of the Pólya's problem solving strategy, which provided further evidence of the students' improvement in problem solving.

## 1. Introduction

Problem solving is the process of reaching an acceptable solution to a novel problem, which usually involves critical thinking and analytical reasoning skills [1, 2]. Mathematics often requires problem solving skills, and it is not an easy task for teachers to cultivate creative mind and to always design lessons that provide supports in developing problem solving skills [3, 4]. One framework for thinking about problem solving was suggested by Pólya [5], and the strategy is recognised by many people as the steps they undergo during problem solving. He suggested the four stages to problem solving:

1. *Understanding the problem.* The first stage involves comprehending the problem by stating it in our own words, identifying the given information and what is being asked.
2. *Devising a plan.* Once the first stage is done, we can now look for the strategies to solve the problem, for example drawing a diagram or using variables to create an equation.
3. *Carrying out the plan.* This stage includes executing the strategy identified in the previous stage. If the strategy does not work, other strategies will be sought until the problem is solved. Most often, this stage includes the process of trial and error.



4. *Looking back at the process.* Look back at the problem to ensure all parts of the questions are answered and all conditions satisfactorily met. Reflecting back at the problem also involves recognising the undergone process and being able to relate it to other future problems.

### *1.1 The thinking aloud pair problem solving strategy*

Stice [6] suggested that students should be given opportunities to work collaboratively and receive constant feedback on how they are doing in order to improve their learning. One pedagogical technique for problem solving that provides equal experience for the whole class is the 'Thinking Aloud Pair Problem Solving' (hereafter, referred to as TAPPS). Claparède [7] first studied TAPPS, which was later expanded by Whimbey and Lochhead [8]. Their idea was to have students paired up, one as a problem solver, and the other as a listener. The problem solver reads the problem aloud, while the listener listens. The problem solver has to keep talking, expressing all his/her thoughts, and the listener has to prompt the problem solver if he/she stops talking, for example by saying "Tell me what you are thinking". The listener also needs to try to understand what the problem solver has done, so he/she should question the problem solver for further explanation (e.g., "Why do you say that?" "I don't understand. Would you explain that to me?" etc.). The listener should keep in mind not to solve the problem or direct the problem solver to the answer by giving hints. The main purpose of TAPPS is not about getting the right answer, but rather understanding how they reached an answer, even if they are incorrect, which may be achieved through verbalising the process [8].

The TAPPS strategy has been used across disciplines at several secondary levels. For instance, for the application of TAPPS on reading comprehension, it was found that TAPPS increased reading comprehension scores in students with poor reading comprehension ability [9], and even for students with negative attitudes towards reading [10]. Similarly in chemistry, it was found that the students who applied the TAPPS strategy showed greater problem solving ability than those students who applied with other strategies, and that they performed better at recalling mathematical concepts, executing procedures, and conceptual knowledge [11]. Jeon et al. [11] also provided evidence to demonstrate the importance of a strong emphasis on the role of the problem solver and the listener. Their study showed that the listeners exhibited more verbal interactions than the problem solvers. For example, when problem solvers exhibited behaviours of 'requiring agreement', 'providing' responses, or 'modifying' the process or response, the listeners 'agreed', 'asked', and 'pointed out' most of the time.

In the context of mathematics, Kani and Shahrill [12], and Irham and Zainuri [13] examined the application of TAPPS for problem solving on rate, ratio and percentages, and algebra respectively. Kani and Shahrill [12] investigated Year 9 students' problem solving behaviour and perceptions of the TAPPS strategy via questionnaires. They found that the students did not significantly improve their mathematics achievement and conceptual knowledge, but with TAPPS, they were helped to be more conscious of their thinking process and to improve their problem solving skills. Further analysis of the problem solving behaviour questionnaire also showed that the students improved most remarkably in 'understanding the questions'. Kani and Shahrill [12] also pointed out that based on the findings from the TAPPS questionnaire, the students had positive perception and attitudes towards the TAPPS strategy, and they overall agreed that TAPPS could help them in the learning of mathematics.

TAPPS involved collaboration between students, and working collaboratively has become a 21<sup>st</sup> century trend [14]. Collaborative learning refers to "an instruction method in which students at various performance levels work together in small groups towards a common goal" [15]. Collaborative learning develops social interaction skills, and through it, students will gradually realise the need for positive and supportive interactions in order for the group to succeed. Along the way, they learn to recognise the behaviours that help them work together with their peers, and to identify and reflect their own contributions [16]. Additionally, students could also learn to identify their differences and find solution to disagreements or any social conflicts that may arise when working together [17]. Ultimately, collaborative learning promotes positive societal responses and increases oral communication skills [18, 19].

### 1.2 Fractions and fractions problem solving

Fractions are one of the essential mathematical concepts that students need to grasp before they can proceed to more advanced mathematics topics [20]. Yet, most students found fractions to be difficult to learn [21, 22]. Some argued that this was due to the confusing relationships between different fractional concepts and arithmetic procedures [23]. For example, the fraction  $\frac{3}{8}$  can be seen as three out of eight equal parts (part-whole), as three parts to eight parts (ratio), as three divided by eight (quotient), as a point on a number line (measure), or as three-eighth of a quantity (operator) [24]. On the contrary, it was argued that the concept of fractions begins as a procedural activity [25, 26]. This was consistent with Kerslake [27] who found that many students could perform addition of fractions correctly, but when asked, they were unable to justify the method that they used, suggesting that there is a need for students to learn fractions both procedurally and conceptually.

The sampled students in a Malaysian study found word problems involving fractions to be more difficult compared to the fraction computational problems [28]. According to the experienced teachers they interviewed, students could not relate their mathematical knowledge to real life situation, which resulted in difficulties with solving word problems. The students performed best in addition and subtraction word problems, and worst in multiplication word problems. Division word problems were slightly higher than multiplication, attributing to the fact that students could identify fraction division problem easier than multiplication division. They also observed that students tended to not write down all the necessary steps to solve a problem using the correct mathematical syntax and grammar. Therefore, they believed that students must be trained to write these steps (i.e. their thinking process), so that problem solving can become easier.

## 2. Method

This study implemented the TAPPS and the Pólya's Problem Solving strategies to support students' learning of fractions. The research question explored was "What differences can be observed in the students' problem solving behaviour before and after the implementation of TAPPS and Pólya's Problem Solving strategies?" An action research framework using a mixed method design was employed with data collected from the pre and post-tests, questionnaires and interviews. A total of 51 mixed ability students (30 male and 21 female) from two Year 7 classes of a government secondary school in Brunei Darussalam took part in this study. At the time of the study, the participants' ages ranged from 11 to 13 years old ( $M = 12.1$ ,  $SD = .392$ ). Permission and the approval to conduct the study were obtained from relevant agencies, and the students and the school were ensured that their participations were kept confidential.

### 2.1 Data collection instruments

The data were collected from the three types of instruments:

1. *Mathematics Achievement Test*. The first author designed this test, which consisted of seven questions: four questions on computational fractions and three questions on fractions word problems. Three experienced teachers were also invited to review the questions to ensure that the test was at an appropriate level of difficulty. The test was used for both of the pre- and post-tests and the duration of the test was 30 minutes.
2. This test consisted of seven questions: four questions on computational fractions and three questions on fractions word problems. The test was used for both the pre- and post-tests and a 30 minutes duration was allowed to complete the test.
3. *Problem Solving Behaviour Questionnaire*. The questionnaire used in this study was adapted from Kani [29]. It was administered to students both before and after the implementation of TAPPS. There were 25 items, and the students were required to rate their answers in a 5-point Likert scale, ranging from 1 (*Never*), 2 (*Seldom*), 3 (*Sometimes*), 4 (*Often*), to 5 (*Always*). All the questionnaire items were based on Pólya's four stages of the problem solving strategy.
4. *Recordings of the lessons and interviews*. Interactions between pairs of students were audio and video recorded in order to analyse students' interactions and problem solving behaviour. In

addition, 17 selected students were interviewed after the post-test to delve further into their problem solving behaviour using the TAPPS strategy. The students were mainly selected for the interviews because they showed great improvement in their pre- to post-test scores, for their notable answers in the questionnaire, and/or for their keen effort during the intervention lessons.

## 2.2 Procedure

The students were given the pre-test and the problem solving behaviour questionnaire at the start of the first lesson, before being introduced to the TAPPS and the Pólya's problem solving strategies. After the administration of the pre-test and the questionnaire, the students were provided with two instruction sheets in order to help them familiarise with the TAPPS and Pólya's problem solving strategies. The first instruction sheet, which was adopted from Stice [6], provided the guidelines for the problem solver and listener, including a step-by-step procedure on the role of the problem solver and the listener, from sitting comfortably before starting the problem solving to how to first approach the problem. The second instruction sheet provided the students with the guidelines for the Pólya's problem solving strategy [5], which instructed them what they can do in each stage of Pólya's problem solving strategy such as rewriting the information given, or by drawing pictures to solve the given problem.

The students were paired up according to their pre-test scores, where the pairings consisted of one high achiever and one middle achiever, as well as pairs of students with average scores. Subsequently, after the pairings, they were encouraged to apply both of the TAPPS and the Pólya's four-step problem solving strategy to solve the first exercise worksheet. The teacher would facilitate the students when they required her assistance, for example, by guiding the students during the process of understanding the problem through step-by-step questioning. The correct answers were also provided during the lessons to guide the students in their problem solving. To guide the students' use of the TAPPS and Pólya's problem solving strategies, the teacher consistently reminded the students to refer to the disseminated guidelines. In total, the students were given two exercise worksheets during the 3-week intervention lessons, and once they have completed both worksheets, the post-test and the problem solving behaviour questionnaire were then administered. All computational fractions questions from the pre- and post-tests, and the exercise worksheets were required to be solved without using a calculator, and calculators were only permitted for word problem questions. The entire data collection process, including the student interviews took approximately three weeks.

## 3. Results and discussion

### 3.1 Differences in students' problem solving behaviour

In order to investigate the differences in students' problem solving behaviour before and after the implementation of TAPPS, the mean scores and standard deviations from the questionnaires (before and after the intervention) were calculated. There was a difference of 6.6, which was then tested for significance level. As the distribution was found to be not normal, the Wilcoxon Signed Rank Test was used instead. According to the test, a significant difference in the student problem solving behaviour was detected,  $z = -3.68$ ,  $p = .000$ , where the mean score for after ( $M = 95.5$ ,  $SD = 13.8$ ) was higher than the mean score for before ( $M = 88.9$ ,  $SD = 15.2$ ) the implementation of TAPPS, as shown in Table 1. It also showed that the students' problem solving ability have improved in the post-test. And the calculation of effect size ( $r = \frac{Z}{\sqrt{N}}$ , where,  $Z = Z\text{-value}$ ,  $N = \text{number of observations i.e. at Time 1 and Time 2}$ ) showed a large effect ( $r = .36$ ).

**Table 1.** Mean scores and standard deviations of problem solving behaviour before and after TAPPS.

( $N = 51$ )	Before	After	$Z$
Problem solving behaviour	88.9 (15.2)	95.5 (13.8)	-3.68*

Note. \* $p < .001$ . Standard deviations appear in parentheses next to the mean score.



To explore the abovementioned differences, the data was further analysed by using the Wilcoxon Signed Rank Test. Items that represented each stage of Pólya's problem solving strategy were summed separately, and the mean scores and standard deviations for each stage are presented in Table 2.

**Table 2.** Mean scores and standard deviations for the four stages of Pólya's problem solving strategy.

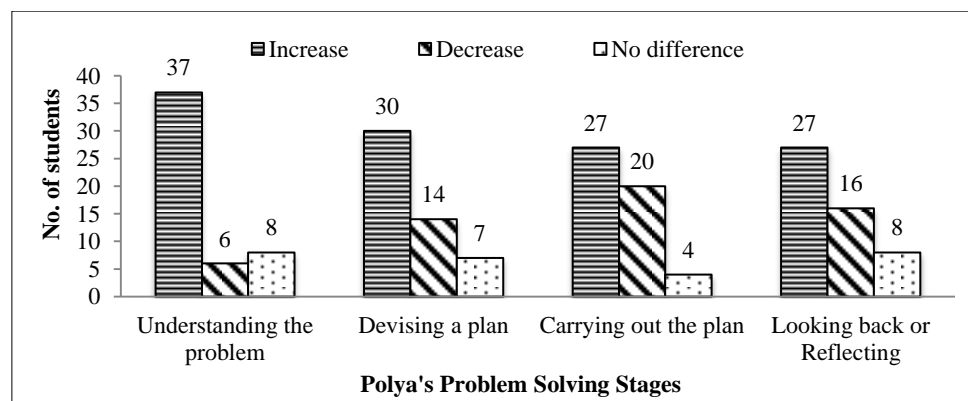
(N = 51)	Pre-test	Post-test	Z
Understanding the problem	27.8 (4.36)	30.9 (4.93)	-5.08*
Devising a plan	13.6 (3.33)	14.8 (2.98)	-2.78**
Carrying out the plan	21.6 (4.94)	22.9 (3.67)	-1.84
Looking back or reflecting	22.0 (5.33)	23.1 (4.04)	-1.46

Note. \* $p < .001$ . \*\* $p < .01$ . Standard deviations appear in parentheses next to the mean score.

The Wilcoxon Signed Rank Test showed that there were statistically significant differences in 'Understanding the problem' and 'Devising a plan' stages, where  $z = -5.08$ ,  $p = .000$ , and  $z = -2.78$ ,  $p = .005$ , respectively. The magnitude of differences in the means was large for both stages, according to the effect size ( $r = .50$  and  $r = .28$ ). The differences in the other two stages did not differ significantly,  $z = -1.84$ ,  $p = .065$  for 'Carrying out the plan' stage and  $z = -1.46$ ,  $p = .143$  for 'Looking back or reflecting' stage. Nevertheless, it was also found that there were slight increases in the test scores from the pre-test to the post-test for these two stages.

### 3.2 Differences in each stage of the Pólya's problem solving strategy

The differences in each stage of Pólya's problem solving strategy were subsequently explored. For a positive impact, the ideal situation would portray a high number of students with increased scores in their problem solving behaviour in the post-test compared to the pre-test. And based on the students' scores obtained from the problem solving behaviour questionnaire in Figure 1, there were more than 27 (out of 51) students whose scores increased in all stages of the Pólya's problem solving strategy.

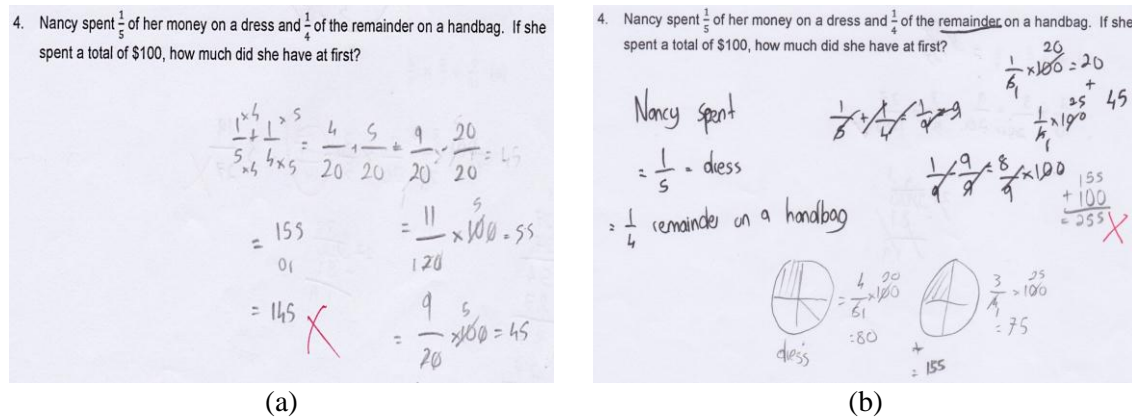


**Figure 1.** Variations in students' problem solving behaviour (N = 51).

#### 3.2.1 Understanding the problem

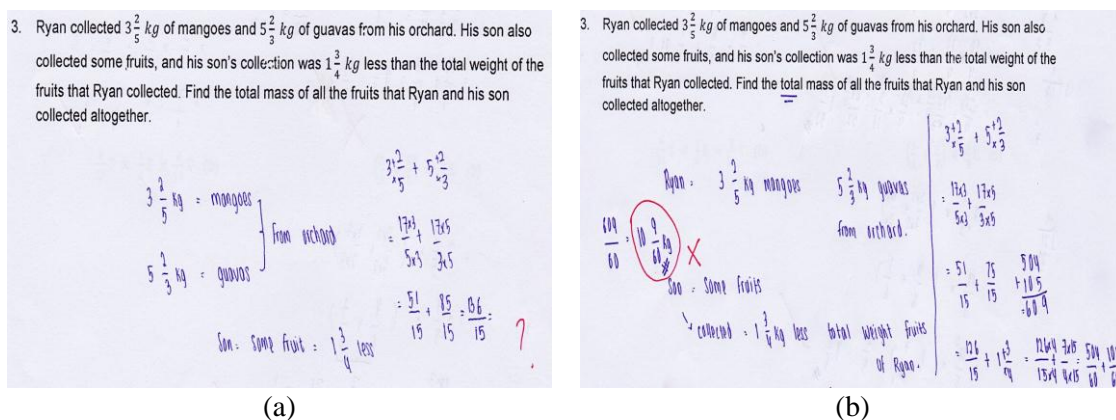
The first stage plays an important role in the process of problem solving, since, if the students could not grasp the problem, they would not be able to proceed with the next three stages. As shown in Figure 1, compared to the other stages, this stage showed the most improvement. For example, there were 37 out of 51 students whose scores increased in the 'Understanding the problem' stage after the intervention, while there were relatively fewer students whose total scores decreased (6 students) or remain unchanged (8 students). Figures 2 and 3 show examples of worked solutions from two students, Farah and Fatin (pseudonyms), respectively. It was noticeable that there were changes in their problem solving behaviour before and after the intervention. Both students could be seen writing

out their thought processes to help them understand the problem, albeit they did not correctly answer it in the end. Specifically, Figure 2 shows that in the pre-test, Farah only showed the calculations when answering Q4, but after the intervention, her method in understanding the problem involved writing out more important information that she extracted from the question.



**Figure 2.** Examples of Farah's worked solutions for the (a) pre-test and (b) post-test.

Fatin's thought processes were seen in both the pre- and the post-tests, as shown in Figure 3. During the interview, she mentioned that she normally would write down what she understood from the questions. It was also worth noting here that compared to the pre-test, her thought processes appeared to be broader and more detailed in the post-test.



**Figure 3.** Examples of Fatin's worked solutions for the (a) pre-test and (b) post-test.

A student's ability in understanding and extracting keywords appeared to determine whether or not they would be able to get the correct answer. This was particularly the case for Q3 and Q4. For instance, this study found that the key to solving Q3 lied on the statements 'less than the total weight' and 'total mass of all the fruits'. As shown in Figure 3, it appeared that Fatin interpreted 'total' by adding up the numbers, but she did not realise what to do with "less than". Meanwhile, the key to understanding Q4 depends on the word 'remainder'. Yet, most students mainly performed operations on  $\frac{1}{5}$ ,  $\frac{1}{4}$  and \$100 in which they did not show an understanding of the statement: ' $\frac{1}{4}$  of the remainder'. For example, in Figure 2, it was seen that Farah added  $\frac{1}{5}$  and  $\frac{1}{4}$  and multiplied by 100 when answering Q4. Furthermore, it was also found during the interviews that most of the interviewed students immediately said ' $\frac{1}{5}$  and  $\frac{1}{4}$ ', when they were asked what the fractions were for Nancy to spend on a dress and a handbag.



### 3.2.2 Devising and carrying out a plan

In this study, it was found that more than 40 students showed attempt to devise and carry out a plan when answering the questions both in the pre- and post-test. Notably, they also appeared to become better at devising and carrying out the plans (refer to Figure 1), regardless whether they actually knew how to solve the problems. This demonstrated that most students have, to some extent, improved in problem solving, although not necessarily in achieving the correct answers.

According to Pólya [5], devising a plan includes recalling similar solved problems, and considering whether the method used could also be applied to another problem, which Hatana [30] referred it to ‘adaptive expertise’—the ability to apply knowledge and experience to solving problems. In this study, we found that for example, in Q3, there were still 38 students who did not answer the question correctly in the post-test, despite solving several questions of similar concepts in the practice worksheets during the course of the intervention. As mentioned in the section 3.2.1, this may be due to the fact that they did not really understand the problem. Furthermore, it also suggested that many students needed help to develop adaptive expertise, since they were unable to recognise the similarities between the questions that they had done before and the questions in the test and thus, failed to apply the knowledge and experience to solve Q3 in the post-test. In particular, there were nine students who simply made attempts to solve the first or second sentence of Q3 and did not execute their solutions until the end. For example, as shown in Figure 3, Fatin’s worked solutions mainly wrote ‘ $3\frac{2}{5} + 5\frac{2}{3}$ ’. Moreover, in the interview, it was found that when the students reviewed their solutions, they realised that they needed to calculate  $\left(3\frac{2}{5} + 5\frac{2}{3} - 1\frac{3}{4}\right)$  to get Ryan’s son’s collection, however, they remained unaware that they should have also added Ryan’s son’s collection with Ryan’s own collection in order to answer Q3 correctly.

### 3.2.3 Looking back or reflecting at the process

As shown in Figure 1, there were 27 students who considered themselves to increase their problem solving behaviour in terms of the last stage of Pólya’s problem solving strategy: ‘looking back or reflecting at the process’. Nevertheless, the analysis of the students’ responses to questions in the pre- and post-tests did not show any indications regarding whether or not they looked back at their worked solutions or calculations, both in computational fractions and in word problems. It was also worth noting that probably because calculators were allowed for the word problems, double-checking with the calculation may be done with calculators rather than written computation on the test papers.

## 3.3 Discussion

The findings of this study showed that out of Pólya’s four stages of problem solving process, the students showed the greatest improvement in the ‘Understanding the problem’ stage, and to some extent, they have also improved in the ‘Devising a plan’ stage. Yet, as Ahmad, Salim and Zainuddin [28] argued, students tend not to write down all the steps of their solutions; neither in computational problems nor in word problems. Similarly, in this study, it was also found that most of students’ development in devising a plan and carrying out the plan could not be seen, since there was not much to be noted down by them on the test papers, especially in word problems where only the final answers were found in most of the students’ written responses. This would perhaps be a consequence in allowing the use of calculators, as mentioned earlier. Nevertheless, our study found a consistent result with the literature that students might not be able to explain and recall how to solve the questions, even when they could give the correct answers [27].

Contrary to the claims made by Irham and Zainuri [13] that students from all ability classes (low, intermediate, and high) were able to extract necessary information from the questions, this seemed not to be entirely true in the present study. We found that while the students were able to identify important information such as fractions, they were not able to extract other equally important keywords or statements. For example, in Q4, “Nancy spent  $\frac{1}{5}$  of her money on a dress and  $\frac{1}{4}$  of the remainder on a handbag. If she spent a total of \$100, how much did she have at first?” the fractions

and the italicised words are equally important in solving the problem. As mentioned in the section 3.2.1, most students were seen either adding or multiplying the two fractions, which they thought that both  $\frac{1}{5}$  and  $\frac{1}{4}$  were spent from Nancy's money. Meanwhile, it was seen that some students would multiply  $\frac{1}{5}$  or  $\frac{1}{4}$  with \$100, indicating that they did not have an understanding of the problem as a whole, which prevented them from devising a plan well.

#### 4. Conclusion

The findings of this study demonstrated that the implementation of the TAPPS strategy was helpful in improving students' mathematics problem solving behaviour. One of the improvements in problem solving was in understanding the problem, where after the intervention, some of the responses by the students became more organised. However, in terms of comprehending the entire question, identifying keywords, and executing their solutions fully were still underdeveloped. Many of the students also could not relate and apply a strategy that they have done to other similar questions during the course of the intervention. Moreover, based on the students' performances in the pre- and post-tests, we also observed several errors that the students commonly made, such as performing incorrect order of operations, under-simplification of answers, incorrect inversion, as well as misconceptions in cancelling common factors and common denominators. This is one of the important implications of this study in which the errors and misconceptions discovered would enable teachers to recognise which aspects of the fractions topic that they need to pay more attention to and teach more thoroughly. The participants in this study were Year 7 students, so identifying their conceptual misunderstandings at an early stage before they move on to advanced mathematics were especially important and advantageous.

The TAPPS and Pólya's problem solving strategies were generally well received by the students, and they were eager to participate in the lessons. Amongst the responses were "Grouping and teamwork is good because it becomes easier to understand when he explains", and "Pólya's strategy is helpful because it tells you step by step to solve problems". The present study provided evidence to show that not only did the TAPPS strategy facilitate students to verbalise and organise their thinking, it was also a great medium for students to engage in collaborative learning by trying to understand their partner's thinking. Both strategies could be efficiently utilised either individually or together, but it is recommended that they be used simultaneously since a combination of both would be more effective [31-34]. And both of these strategies could also be used to diversify the teaching of fractions. We believe that if students consistently continue their effort and use both the TAPPS and Pólya's problem solving strategies, they can be helped to eventually become better at problem solving in fractions in particular, and mathematical knowledge in general.

The limitations of this study lie in the intervention lessons. Firstly, there were challenges in conducting these lessons. In the beginning of the first lesson, the students were provided with a detailed guideline on the roles of the problem solver and listener, and they were given explanations on what they can and cannot do as a problem solver or listener. This was repeated continuously in most lessons. Yet, not many students actually followed the rules. The problem solvers tend to exhibit behaviours such as 'requiring agreement', while the listeners 'agreed', 'pointed out', and also to the extent of giving hints to the problem solvers [11]. When tested on what the students did when they were the problem solver and listener, two out of four students understood their roles precisely. They explained that when they were the problem solver, they must say aloud everything and teach the listener unless they no longer know how to explain, and if they were the listener, they must "sit tight, be quiet, and listen" until the problem solver encountered any problems. However, the other two students had some misunderstandings. One student reported to have given some hints and pointed at mistakes when he was the listener, which in fact he should not have done so, while another thought that when he was the problem solver, he should ask if there was anything he could not understand. Therefore, it is recommended that future researches could use visual or physical demonstrations before conducting the lessons, for example, with video demonstrations or through role-play.

Secondly, some pairings might not be as effective as the others [35, 36]. In order to get the most out of the TAPPS and Pólya's problem solving strategies, the students abilities in each pair need to be balanced, so one had a higher score than the other (with a difference of two or three marks between them). However, since there was more number of students whose scores were average, there were also pairs where both had the same scores. Genders and existing seating arrangements were also taken into account. In both classes, the boys and girls were seated separately. In order to minimise movements and unfamiliarity between the students, the girls were paired up with girls, and boys with boys (with the exception of two pairs). During the lessons however, both students in several of the pairings still worked on the problems individually even after changing partners and prompts. Therefore, it is suggested that for each question or lesson, the pairings could also be rotated to allow collaborative partnership in learning.

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